Flexibility and Rigidity of Cross-Linked Straight Fibrils under Axial Motion Constraints

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Abstract: The straight fibrils are stiff rod-like filaments and play a significant role in cellular processes as structural stability and intracellular transport. Introducing a 3D mechanical model for the motion of braced cylindrical fibrils under axial motion constraint; we provide some mechanism and a graph theoretical model for fibril structures and give the characterization of the flexibility and the rigidity of this bar-and-joint spatial framework. The connectedness and the circles of the bracing graph characterize the flexibility of these structures. In this paper, we focus on the kinematical properties of hierarchical levels of fibrils and evaluate the number of the bracing elements for the rigidity and its computational complexity. The presented model is a good characterization of the frameworks of bio-fibrils such as microtubules, cellulose, which inspired this work.

Keywords: fibrils, cross-links, flexibility, rigidity, optimization, graph connectivity

Introduction

The fibril network literature declares that the cross-linked fibril structure is complicated and consist of redundant connections [34, 42, 29, 5]. These redundancies are necessary the loading, the signal transporting, and the stability point of view. However, technical systems or the structures of the natural or social sciences have to be rigid and/or flexible some case at the same time [36, 11, 19, 30, 17, 15]. This paper characterizes the rigidity and mobility of rod-like fibrils in biological systems. The rod-like fibril structure with redundant bracing elements is safe if some of the bracing elements would collapse than the remainders make the structure rigid yet.Celluloses [24, 25, 63, 55, 10, 43, 66, 64], Fibrin [14], Collagens, Minerals, Microtubules,[71, 8, 18, 56, 39, 38, 41, 49, 74, 28, 1, 68],Chitins [58, 61, 62, 33] self-assemble into thick, hierarchically ordered, stiff fibers through electrostatic and hydrophobic interactions,. The network stiffness becomes surprisingly insensitive to network concentration, demonstrate how a simple model for networks of elastic fibres can quantitatively account for the mechanics of reconstituted collagen networks. We provide a discrete model that characterizes the flexibility and rigidity of braced framework of the fibrils. The paper gives a model for further improvements.

1.1. Bar-and-Joint Framework

The bar-joint framework: One of the simplest structures in statics is the bar-and-joint framework, that consists of optimal bars connected by rotatable joints, i.e. the bar lengths and the bar and joint incidences must be preserved.

1.1.1 The rigidity of bar-and-joint framework

Firstly, we give a definition of the rigidity of the bar-and-joint framework.

Definition 1: A framework is rigid if any continuous motion of the joints that keeps the length of every bar fixed, also keeps the distance fixed between every pair of joints.

It is a preservation of distances the joints during any continuous motion of the joints.

1.1.2 The infinitesimal rigidity of bar-joint framework

The infinitesimal motion is a special case of virtual motion; that refers to a virtual change in the position such that the constraints remain satisfied (possible motions).

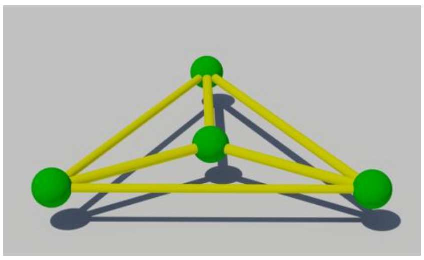

Figure 1

Infinitesimal motion: There are non-trivial infinitesimal motions of the central joints of the framework. The central points of its joints are in the same plane. The central joint can move infinitesimally into the normal direction of framework plane, at the beginning of this motion, no constraint restrict these motions. The length of the shorter bars change in second-order, the preservation of all distances is first order.

The rigid body motions refer to the trivial infinitesimal motion. The Definition 1 allows frameworks that have infinitesimal motions. In the statics for the rigid structure, the non-trivial infinitesimal motions of the joints have not permitted [67, 54, 60, 57, 51, 59, 65, 40].

Definition 2: A framework is infinitesimally rigid if it only has trivial infinitesimal motions.

We can see the central joints of the framework on Figure 1. It has an infinitesimal motion up and down, perpendicularly to the plane of the structure. If a framework is infinitesimally rigid, we require first-order preservation of distances during the infinitesimal motions of all joints. We have to decide the rank of rigidity matrix of the framework.

Let (x_i,y_i,z_i) be the coordinates of the joint Pi of a bar-joint structure, where $(1 \leq i \leq n)$. A bar between the joints P_i and P_j determines the distance from P_i to P_j, there for it is constant, by differentiating its square, leads to the next equation:

$$(x_i - x_j)(\dot{x}_i - \dot{x}_j) + (y_i - y_j)(\dot{y}_i - \dot{y}_j) + (z_i - z_j)(\dot{z}_i - \dot{z}_j) = 0 \quad (1).$$

Where, the velocity coordinates $\dot{x}_i$, $\dot{y}_i$, $\dot{z}_i$ are the varieties. Hence, if we use bars between joints, and the number of bars is e, then we get a system with e pieces of equations. The matrix representation of the equation system is the next:

$$Au = 0 \qquad (2).$$

Where u is the column vector of velocity, and A is an $e \times 3n$ rigidity matrix. In statics, rigidity does not even allow infinitesimal motions. In this case, the equation (2) has the trivial solution only (i.e. the rigid body like motions). The rigid body motion of the joint keeps fixed the distance between the pairs of the joint. If the joints of the framework have motions, that different from the rigid body motion, then the framework is not rigid. In this case the $rank(A)<3n-6$. The framework is rigid if and only if the $rank(A)=3n-6$, see: [47, 67, 60, 70, 50,37, 54, 57]. Maxwell (1864) gave this characterization but with his result, the time complexity of deciding the rigidity is $O(e^3)$.

From this point, the infinitesimal motion and the infinitesimal rigidity refers to motion and rigidity respectively.

1.1.3 Some bar-joint framework that has better rigidity characterization

We want to characterize the rigidity in a simpler way. Such characterizing has known to square grids and annex building and many other structures.

It has shown that the rigidity of the structure depends on the places of the bracing elements rather than on their number. The geometric methods can be very efficient but are only applicable to those parts of the problems where the places of the joints are dependent. There are interesting results for lattices, tessellation, and other repetitive structures, see: [6, 3, 60, 50, 20, 21, 59, 51, 27, 32, 7, 48, 65, 46, 57, 12].

The mentioned results are better from an algorithmic point of view, because the size of the bracing graph of the framework, and hence the time complexity of the proposed algorithm, is smaller then consider the rank of the rigidity matrix.

2. Bracing Straight Fibril

2.1. The Structure of Straight Fibril

This paper gives a graph theoretical characterization of the rigidity of right circular cylinders with equal radius and different heights, where heights are significantly greater, then the radius. These cylinders can connect with each other by bracing elements, on their surfaces; these connections firstly substitute by bars, later we will use other type bracing elements also. The bracing element refers BE. The cylinders correspond to Straight Fibrils, referred to as Straight Fibrils, SF because they can move back and forth only; these are 1D segments, they can move on lines in 3D the consequence of the axial motion constraint. We take a sequence of Straight Fibrils that are in the axis of the cylinder, and the distance of neighbouring joints, they connect the Straight Fibrils are equal to the distance between the neighbouring bracing elements along the axis of the cylinder. If the coaxial cylinders connect to each other with their full base faces, then the corresponding Straight Fibrils are attached. We can see four cylinders on the left in Figure 2 and the corresponding SF framework on the middle of the figure. If there are some bracing bars between the joints of two SFs, we refer to them as BEs or bars. We visualize our structures as 3D, effectively in our model, the joints are zero-dimensional, the braces and Straight Fibrils are 1D.

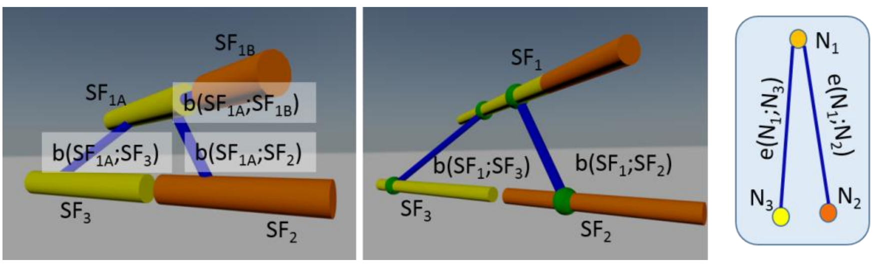

Figure 2

We illustrate the correspondences from fibrils structure to its graph across the SF framework. We can see four yellow cylinders on the left, the uppers connect with the base face with blue links, and the corresponding two Straight Fibrils are sticking together in the middle part of the figure. Two blue cross-links are connecting both under cylinders with the upper left ones on the left part of the figure, the intersection of cylinders and cross-links have corresponded with ball joints. The bracing graph of the framework is to the right of the figure.

Having applied the rigidity theory for this unique bar-joint structure we give some necessary and sufficient condition for the rigidity of this structure that consists of straight fibrils and their movement under direction constraint. Combinatorial characterizing is not known to straight fibril at direction constraint in 3D space, however in (Recski 1989), frameworks were discussed, in which joints placed to horizontal or vertical tracks (i.e. they can move only in the prescribed direction) in the plane. In this paper, we characterize the motion of the fibrils in 3D under direction constraint, and the directions are arbitrary.

There is no buckling the consequence of the motion constraint. The lengths of the Straight Fibrils are arbitrary in SF. The SFs and the BEs assembled a framework. We will demonstrate that if there are enough BEs in the right positions than the framework will be rigid.

Definition 3: The SF framework consists of SFs and some BEs that link them together.

We refer the SF framework as SFF.

2.1.1 The motions under direction constraints

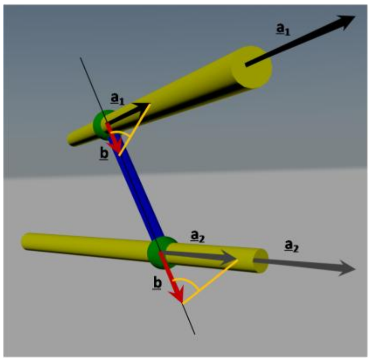

Figure 3

We can see two Straight Fibrils connected by abar in skew position. The orthogonal projections of the motion vectors of the Straight Fibrils to the axis of the bar are equal the consequence of the rigidity of the bar.

SF as a rigid bar under direction constraints can only move to the same direction as its axis. We will use a bar as a BE between two SF-s, they are under direction constraint.

Theorem 1: The BE is not perpendicular to neither of the two axes of the Straight Fibrils, if and only if both Straight Fibrils have infinitesimal motion (not zero), or none of the Straight Fibrils have any.

Proof: (by contradictions). If the SFs are parallel, then those infinitesimal motion components are equal. Otherwise, their projections to the axes of the cross-linked bar are different therefore the BE is not rigid.

If the SFs are not parallel, then their infinitesimal motion components are not equal. However, it is not possible that one of them is not zero, and the other is zero. In this case, their projection to the axis of the cross-linked bar are different, see

Figure 3, one of them is not zero, and the other one is zero. Hence, the cross-linked bar could not be rigid.■

Comment 1. If the BE is perpendicular to one of the SF-s, then the infinitesimal motions of the SF-s are independent of the other. In this case, the other has not any infinitesimal motion, if it is not perpendicular to the brace. If the BE perpendicular both of the Straight Fibrils, then they can move independently of each other. It is important that the bars do not connect to the Straight Fibrils at a right angle; in the opposite case, the brace refers to degenerate.

Comment 2. If the bracing bar is not perpendicular to any of their Straight Fibrils, then an infinitesimal motion of one of the Straight Fibril, generate the infinitesimal motion of the other Straight Fibril.

2.1.2 The Bracing Graph

Definition 5. $G_{SFF}(N_i,E)$ is the bracing graph of SFF framework, where N_i, correspond to SF_i; and edge N_i,N_j is an element of E if there is a non-degenerate BE between SF_i and SF_j.

On the right side of Figure 2, we can see the bracing graph of the braced framework that we can see on the middle part of this figure.

2.2. The Flexibility in the Bundle.

We assume that all of the SFs in the SFF are parallel. The SF can move to the direction of its axis, and the BEs restrict the displacements of the SFs.

Fit a coordinate system to the structure, that axis Y will be parallel to the SFs. If one of the bar end points has a motion or an infinitesimal motion in the Y-axis direction, then the other end point of this bar has the same motion or infinitesimal motion in the same direction due to the rigidity of the bars. There is not an X or Z motion component of the joint because the bars can only move into the Y direction, which is the consequence of the direction constraints condition.

Denote by SF_i; the element i-th of the set of the SFs and denote by yi the infinitesimal motion of the Y coordinates of SF_i. Denote by $b(SF_i;SF_j)$ the bracing bar between Straight Fibrils SF_i and SF_j. If there is a brace $b(SF_i;SF_j)$ than $y_i = y_j$. It is important that the brace is non-perpendicular to the Y-axis; in that case, it is possible that the infinitesimal motions of its endpoints are not equal in the direction Y; in this case, the bar is degenerate.

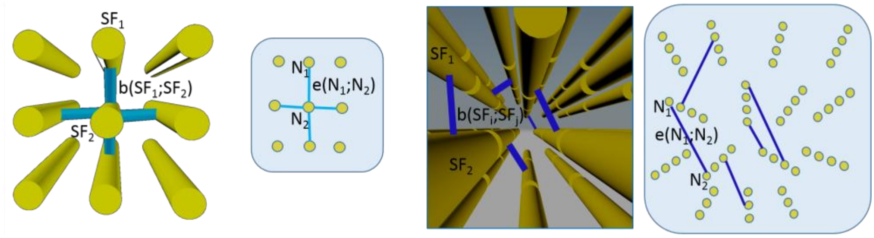

Figure 4

Two type of SFF in Bundle is on the figure the BEs connect the nearest neighbors. On the left part of the figure, we can see the SF-s in square grid positions and triangular grid position on the right. The bracing graph of the frameworks is on the right side of the cartoon of the braced framework. If we project the Straight Fibrils of the framework and their BEs to the Y direction, then we get a structure that is isomorphic to the bracing graph. In the arrangement of the triangular grid, the Straight Fibrils have not bracing links they are on the same axis. They do not compose a long Straight Fibril. Hence, each SF has a node in the graph on the right side.

2.2.2 The Rigidity of the SFF framework in Bundle

Theorem 2: The parallel Straight Fibril Framework in Bundle is rigid, if and only if its bracing graph $G_{SFF}(N_i,E)$ is connected.

Proof: If the bracing graph is connected then $y_i = y_j$ to each SF. Hence, they can move each other only into the direction constraint. Hence, they have the only motion of the translation that is a rigid body motion.

If the bracing graph is not connected then there exists an index j so that we cannot get from node N_1 to node N_i along edges in the bracing graph, hence the $y_1 \neq y_j$.Therefore, SF_1 can move without reference to SF_i.■

Comment 3. The motion or the infinitesimal motion of one of the Straight Fibril travels along not degenerated bars to the other Straight Fibrils they are in the same corresponding components of the bracing graph. The displacement of the Straight Fibrils does not change if they are connected with BEs.

Comment 4. Accordance with the Definition 2 the SFF in Bundle is rigid; apparently, all fibrils in the bundle could move in its matrix fluid independently the number of the BEs. The drag of the BEs obstructs this traffic, partially; and the BEs of the hierarchical structure of the bundle that is also fibril, resist this kind of motions.

Next, we consider two types of connection by BEs: in the first case, there are some bracing bars in the SFF, and the second case, snap bracing, we discuss the two instances in the different arrangement of SF.

3. SFs in Layers with different bracing connections

Up to this point, we do not assume that the SF-s connect to the nearest neighbour, nor the regular arrangement of them, we also give up that the all of the SF are parallel in the framework, and we choose to connect nearest neighbours further. We suppose that axes of some SF are parallel, and its axes are in the same plane. They compose a layer.

3.1. The rigidity of SF-s in one layer with regular BEs

3.1.1 The rigidity of SFs in one layer with regular BEs

Firstly we consider the case when the SFs connect by regular BEs to the other SFs that would be in the same layer (first case), or will strictly be in another neighbouring layer (second case). In both cases, the BEs are not degenerate. We discussed the first instance since the second is special of its.

We suppose the directions of the fibrils that are in different layers are not parallel. Therefore, the SF can connect the all of the SF that are in the neighbouring layers. Hence, the average number of the nearest neighbour of one fibril will be more than the structure that was considered earlier, where the average number of the nearest neighbour is not more than six.

3.1.2 The bracing graph of SFs in layer with regular BEs

According to the earlier bracing graph definition $G_{SFF}(Ni;E)$ is the bracing graph our SF structure, where Ni, correspond to SFi; and edge Ni,Nj is the element of E if there is non-degenerate BE between SFi; SFj. On Figure 5, we can see the bracing graph of the braced SFs in the layer.

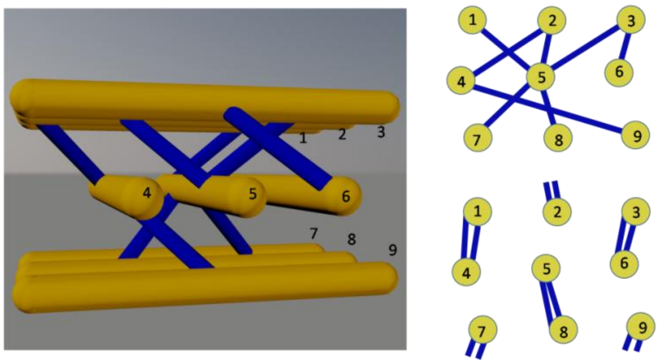

Figure 5

An SFF is arranged into layers. On the left, we can see an SF framework in layer structure; the joints are not signed. We can see on the right above the bracing graph $G_{SFF}(N_i;E)$ of the SF framework on the left side. A part of the bracing graph of a potentially braced framework by snaps is shown on the right below.

Using Theorem 1, we could get to the next false theorem for SF framework in layer structure:

False theorem 3: The SF layer structure framework connecting by not degenerate bracing bars is rigid, if and only if, the bracing graph of them is connected. The simple connecting of the bracing graph implies the motions of all SF-s, or zero displacements of all SF-s, the consequence of Theorem 1. Take a pyramid for counter example. Let the four SF-s be on the four side edges. Connected them by four bracing bars, we experience that the symmetric cases are mechanism while the asymmetrical are rigid [54, 57, 35].

The next statement is also false: If the SF layer structure framework connecting by not degenerate bracing bars is rigid, than the bracing graph of them is connected. See Theorem 4.

Therefore, we have to modify the definition of the bracing graph for a proper characterization of SF framework in a layer structure, and we have to change the prohibition of the degenerate BEs.

Definition 6: The BEs of the SF framework are generic if any not zero infinitesimal motion of any SF imply the not zero and not equal infinitesimal motion of any other not parallel SF along by BEs.

Definition 7: $G(N_i;E)$ is the bracing graph our SF structure, where N_i, correspond to SF_i and edge N_i,N_j belongs to E with multiplicity, if the BE between SF_i and SF_j, belong to the set of generic BEs of SF framework.

Theorem 4: The SFF as layer structure framework connecting by generic bracing bars is rigid, if and only if all connected components of its bracing graph includes, at least, one circle circuit.

Proof: Along the elements of one of the cycle of the circuit the circle is formed a rigid structure the consequence of Definition 7, hence the connected part of its component is also rigid the consequence of Theorem 1.

If one of the components of the bracing graph does not contain least one circle, then the SFs corresponding to this component can move together the consequence of the Comment 2. ■

Comment 5. If the bracing graph is circle free therefore tree graph, hence we get from node N_1 of the graph to the node N_2 along connected edges. The infinitesimal motion of the corresponding Straight Fibril SF_1 travels connecting with non-degenerated bars to the other Straight Fibrils of the corresponding nodes of the tree graph and get to N_2. The infinitesimal motion of the Straight Fibril may change along the not degenerated BEs, but any of them is not zero.

3.2. The rigidity of SF-s with snap BEs

3.2.1 The rigidity of SF-s with snap bracing

We consider the case when two SFs touch each other in a snap point. This touching point refers Snap; this fixed both fibrils if they are not parallel, and they are under the direction constraints. If they are parallel, then they can move together to them direction. In the bar-joint structure, On Figure 46, we can see a Snap. The BEs connect the joints of bars they have corresponded the touching fibrils. The length of these BEs is 2r, where r is the radius of the cylinder type fibrils. We referred it a short brace. In the case of short brace the requirement of infinitesimal rigidity is not satisfied, the infinitesimal motion is allowed by the short brace to the direction of the axes of the bars. Hence, we have two possibilities, firstly correspond only one joint to the short brace and their two joints; in this case, we have to change the original geometry (the distance of the SF would be zero). Secondly, we have to apply two extra bars, connected the joints of the short brace to the one of the joint that connect the other end point of the short brace, see the bottom of Figure 6. Secondly, we have to apply two extra bars instead of short brace, connected the joints of the short brace of one of the SF to another new joint of the other SF that different from the other joint of the Short brace, see the bottom of Figure 6 with dashed lines. We give similar situation if we use the short brace and an arbitrary brace that not connect to it. It is also enough if we use any two different braces between the two fibrils.

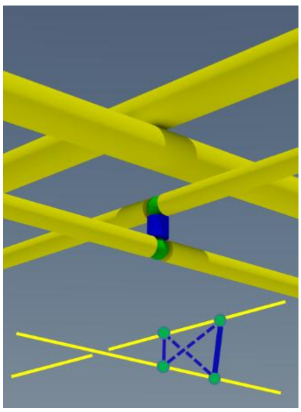

Figure 6

We can see on top of figure two not parallel SF with a snap. They cannot move to the directions of the axes of the fibrils. In the middle part of the picture, a possible bar-joint framework is shown that not suitable about the infinitesimal motions. Bellow, we can see a two possible SF framework that available one of with the dashed lines bars and the other with the solid lines bars.

If two SFs are in different directions, and they have a mutual snap, then they are fixed to each other.

Theorem 5: The SF-s connecting snap to each other is rigid if and only if any of them connected parts contain least two different directed fibrils.

Proof: If the directions of some of the fibrils are parallel than they could move, into that direction, but least one SF connects least one of the others, and they are not parallel. Hence, these two snapped not parallel SFs fix the system. ■

Comments 6. Only the isolated fibril can move in this structure.

On the right-hand side below in Figure 4, we can see a part of a bracing graph of an SFF by snaps that show the number of the snap elements is nearly half of the number of the formal bracing bar elements for the rigidity.

3.2.1 The rigidity of SF-s in with mixed BEs

Both types of BEs could connect two neighbouring SF in the SFF these structures refer SFF with mixed BEs.

Definition: $G_{mix}(N_i;E)$ is the bracing graph of SF-s in layer structure where are snaps connection and regular bracing bars connections mixed. N_i, correspond to SF_i and edge N_i,N_j belongs to E with multiplicity m if there are m pieces of a regular BE between SF_i and SF_j, and edge N_i,N_j belongs to E with multiplicity two if there is snap element between SF_i and SF_j.

Theorem 6. The SFF with mixed BEs is rigid, if and only if, all of the connected components of its bracing graph include at least one circle.

The proof is similar to the proof of Theorem 4.

Comment 7. This theorem characterizes the flexibility and the rigidity of SFs in layers with mixed BEs.

The SFs in a layer structure can connect almost all of the SFs that are in the neighbouring layers if the angle of the SFs in the neighbouring layers is large enough. Hence, the average number of the nearest neighbour of one fibril in the layer structure will be more than the bundle structure where the average number of the nearest neighbour is not more than six. If an SF connects *n* pieces of BEs, then the layer structure are more cohesive than the bundle structure. In the case of removing, one SF from the structures is imperceptible, but in the case of removing some assigned neighboring SFs from the bundle is lighter, because a significant part of the BEs connects the assigned neighboring SFs to each other, contrary to the layer structure. Hence, the SFs rather burst than slip out from the layer structure, seeing the Figure 7, similarly the pictures in [62]

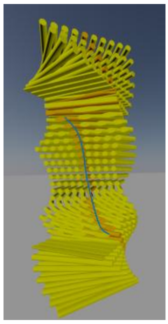

Figure 7

We can see the microstructure of fibrils in layer arrangements. Schematic depiction of the layer structure of the fibrils (seeing the curves) gives similar arrangement in hard tissues [61,62]. Cross fracture of fibril structure mineralized cuticle from the lobster Homarus americanus shows a 180° rotation of ”vertical” fiber layers around the normal direction.

In the case of 3×3 SFF square grid, the number of the BEs that connects them to the bundle structure is nearly 12n; contrary to the layer structure, it is easy to approximate 24n. Intrinsically the proportion of the fixing BEs is approximately twice in a favour of the layer structure.

3.3. A generic structure

The rigidity of generic structure was considered in [2, 45, 60, 70,16, 37, 46, 35]. On Figure 8, we can see an SFF structure if the BEs are not generic, i.e., for instance, *b* parallel *c* than the SF4 can move up or down independently of the others. In the case of generic bracing is out of the question, seeing Theorem 4. The dimension of possibilities when bar b is parallel to bar c, is zero-dimensional. Hence, this case is irrelevant to the possibilities, when they are not parallel. Our structure is generic if the set of coordinates $(x_i, y_i, z_i, X_j, Y_j, Z_j\!: J_i \in J, SF_j \in SFF)$ are algebraically independent over Q (rational numbers).The J signs the set of the ball joint, that connects the BEs to the SFs, and x_i, y_i, z_i is the coordinate of the ball joint J_i [2, 45, 60, 54, 57, 35]. The X_j, Y_j, Z_j there are the coordinates of one of the direction vector of Straight Fibril SF_j. This definition precludes the parallel SFs from the generic SFF. In the Definition 6 was sufficient to prevent the case of the bars that connect parallel SFs.

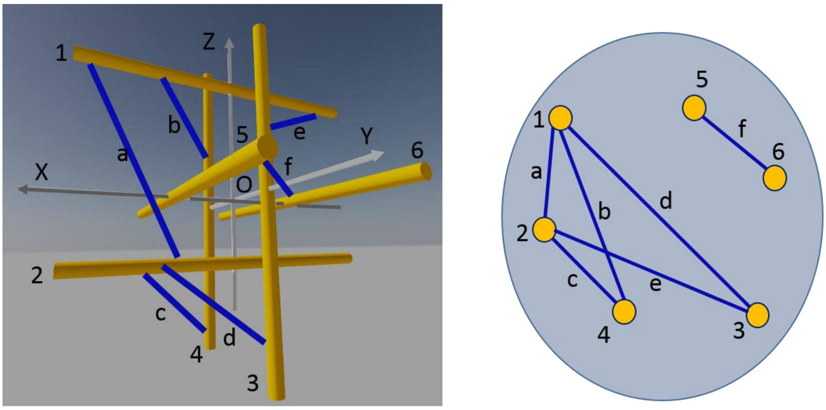

Figure 8

We can see a generic structure of SFF. On the right of the figure, we can see six fibrils signing by numbers, they are almost parallel to the coordinate axes in a couple, they can only move in the directions of the corresponding self-axis without BEs. Using the alphabetic signing BEs will they be flexible or not? We can see on the left its bracing graph. It has a component (*G(5,6;f)*) that not includes a circle. The Straight Fibrils *5* and *6*, with bar *f*, can move together into them self-axis they show in the proximal direction of the axis *Y*; the others fix each other with them BEs.

Our result provides a very fast method for the decision of flexibility or the decision of rigidity of a given of SFF. This novel computational procedure based on the next procedure:

1. Given a fibril structure with BEs.

2. Build the bracing graph of the structure.

3. Is the bracing graph connected?

Hence, the decision of rigidity of SFF originates in the decision of graph connectivity. The complexity of the decision problem of the rigidity of SFF in the case of n pieces of SF and m pieces of BE using Maxwell's result from 1864 is $O((m+n)^3)$ if we use Gaussian elimination for the rank of the rigidity matrix. For the adjacency list representation of the graph, the total time of the algorithm is $O(m+n)$ as its space is also. Accordingly the rigidity of the Straight Fibril is approximately a $(n+m)^2$ times faster than the simulations based on Maxwell' result, hence, the simulated numbers of fibril approximately increase from some hundred to some million.

3.4. The safety of SFs

In the literature, of real fibril declare that the network of the fibril consists of redundant connections. These redundancies are important the point of view the loading and the safety of the rigidity of the network. The fibril structure is asafety if some of the BEs collapse while the remainders have kept the rigid structure yet, i.e. its bracing graph satisfies the corresponding criteria i.e. the connected components include circles. Similar problems are significant and well-studied optimization problems in graph theory, and network analysis. We introduce the case of the SFF in the bundle.

The connectivity augmentation problem is the next in our case: given a bracing graph and a positive integer k, we find a minimum number of new edges in the bracing graph, that the results will be k-node-connected and/or k-edge-connected bracing graph. Hence, if we remove arbitrary k−1 nodes or edges the remained object not collapsed. The connectivity augmentation helps us to increase the safety of an already existing network by adding an optimal number of new connections for the rigidity of SFF.

It is an open question even if the graph G to be augmented is (k-1) vertex-connected. Polynomial algorithms have been developed only for k = 2, 3, 4 by Eswaran [13], Watanabe [69] and Hsu [31], respectively. In network design, it is often of interest to know how sensitive a particular property of a network is to changes in the graph structure, like the removal or failure of edges. We focus on the edge-connectivity of a graph. The connectivity interdiction asks to decrease the edge-connectivity of a graph maximally by removing a limited set of edges. We can ask how many braces could be taken away at most before the graph disconnects. Hence, our framework would collapse. If the edge connectivity is k then we could take away k brace, the graph will become disconnected (not all of k brace are good for the disconnectivity), i.e. the framework will be a mechanism the consequence of the Comment 3. The connectivity interdiction is a reciprocal problem of edge-connectivity augmenting of a graph the weighted optimization problems have been studied in [26, 72].

4. The boundary of the SFF flexibility and rigidity

Our graph theoretical model can determine the flexibility or rigidity the structure of SFs. Maxwell’s result counts the degree of freedom of the framework, and it solves the problem without the direction constraint also, our result for this case is not compatible.

It is expected that overall stiffness of a network consisting of rigid and soft elements be determined mainly by the elastic elements, which are the cross-linkers. In this case, the network stiffness would be insensitive to slight changes in the rigidity of the stiff elements that would be for instant the actin filaments. Structurally related hierarchical tissue systems exist throughout the human body, for example in partially mineralized tissues at tendon-to-bone attachments consisting of collagen cross-linked by stiff mineral particles [43, 22, 44, 4]. Here, the cross-links between fibrils are stiff, and the fibrils are viscoelastic, but analogous dissipative processes and randomness might be important for optimal toughening [53, 40]. This further suggests new principles for the design of synthetic fibril networks with collagen-like properties, as well as a mechanism for the control of the mechanics of such system. With a high resistance to bending and a stiffness of 2 GPa, the microtubules are undoubtedly the strongest cytoskeletal filaments in eukaryotic cells. As such, they play a unique role in some cellular processes, maintaining structural stability and providing a path for intracellular transport [9]. The microtubule structure is similar in all of the eukaryotic cells; it is harder justifiable that the assumption the motions of the fibril are under axial motion control.

Last we can see a multi-layered arrangement of cellulose microfibril at the cell wall surface from onion scale, visualized the microfibril in the surface layer. Note that microfibril merges into and out of regions of close contract. Hence, the direction constraint of the motion of the fibril it is not satisfied. The Artistic Image of the onion cell wall visualized from atomic force microscopy [73]. If we make the graph with one nodes on the fibril, and edges between the nods in case adjacency of the fibril then this graph is not connected. Hence, the connected component of the fibril can move independently each other. In this case, the assumptions of the directed motions and parallel fibrils are not satisfied.

Our results provide strategies that maximize the number of the element of fragments and minimize preservation of intact cellulosic fibrils if the assumption is satisfied for instant in case of arboreal plants. To decrease lignin content while the skeleton of the plant is remaining rigid, we get higher sugar yield.

However, we hold, this model could be a good candidate for the future work in cell biology. Using the results of flexibility and rigidity of SFF, we can predict the motion of a structure of a million individual fibrils thanks to the reduction of the complex system.

Conclusions

We introduce a graph theoretical model for the flexibility and rigidity of the straight-line fibril with bracing elements (BEs) structures under direction motion constraint was referred as Straight Fibril Framework (SFF).

The connectivity and the circles of the bracing graph of the SFF characterize the flexibility and the rigidity of the structure using generic BEs in the most general cases.

We could estimate using this method how many BEs sufficient for the rigidity of the bounds of the fibril. Our results provide strategies that evaluate the number of the BEs of fragments and minimize preservation of intact fibrils.

The kinematical characterization taken provides a powerful model for exploring the mobility of these structures and gives a new approach to understanding the nanoscale behaviour of fibrils.

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